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Magnetization of a clean quantum dot in a time-dependent flux: restoration of Aharonov–Bohm oscillations by inelastic scattering

E N Bogachek†‡, Yu M Galperin§, M Jonson†, R I Shekhter† and T Swahn†

† Department of Applied Physics, Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden

‡ School of Physics, Georgia Institute of Technology, Atlanta, GA 30332, USA

§ Department of Physics, University of Oslo, PO Box 1048, Blindern 0316, Oslo, Norway, and A F Ioffe Institute, 194021 St Petersburg, Russia

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Abstract. We discuss criteria for discriminating between 'ballistic' and 'clean' quantum structures created in the 2D electron gas of gated semiconductor heterostructures and show that they are different for impurity potential fluctuations of short and long range. For long-range fluctuations, such as in high-mobility modulation-doped heterostructures, we show that the criterion for the system to be clean—so that impurity scattering is not at all important—can be met. Hence, disorder-induced gaps in the electron spectrum can be ignored while considering dynamical properties in a time-dependent magnetic field even at a relatively low rate of change. The magnetization of a 2D clean quantum dot turns out to be very sensitive to inelastic relaxation processes in the system. In contrast to the usual destructive role played by inelastic scattering in mesoscopic phenomena, here inelastic scattering *restores* an Aharonov–Bohm type of quantum oscillation in the magnetization. In the absence of such relaxation, strong non-equilibrium behaviour suppresses these oscillations in favour of large diamagnetic moments. We discuss the special type of inelastic backscattering responsible for relaxation in the case of an isolated dot. By monitoring the transient behaviour of the induced magnetic moment as the magnetic field is switched from one value to another we propose to measure the characteristic time of inelastic backscattering estimated to be of the order of 10^{-8}–10^{-7} seconds.

1. Introduction

Magnetic properties of small conductors have been extensively discussed over the last few years (see [1, 2] and references therein). It has become understood that the magnetic moment (and the associated persistent current) induced by an external magnetic flux is a clear manifestation of mesoscopic behaviour.

While persistent currents were originally predicted to appear in clean one-dimensional metallic rings [3], most of the recent discussion has focused on metallic rings containing impurities [4, 5]. The impurity scattering leads to an important qualitative effect—it produces 'forbidden gaps' in the spectrum of quantized electron energy levels in the loop when plotted as a function of magnetic flux. As a result, the single-electron energies are dependent on the flux in an oscillatory way, and dynamical oscillations of the circulating current induced by a quasi-stationary magnetic flux appear. Static magnetic properties of small rings and dots have been discussed by several authors [6–12].

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Non-stationary magnetic responses have been studied much less, the discussions being mostly concentrated on the systems with strong elastic scattering [13–17]. In such systems the most important feature is Landau–Zener tunnelling through disorder-induced gaps in the electron spectrum. Consequently, the problem is connected with the energy level statistics, the results being very sensitive to whether the chemical potential or the number of electrons is kept constant.

Experimental techniques are available for producing laterally constrained two-dimensional (2D) electron systems in gated semiconductor heterostructures where the electron mean free path ℓ is greater than the system's size R. In such pure systems the transport is ballistic and its main features are only weakly influenced by scattering effects [18]. Recent experiments provide convincing evidence of Aharonov–Bohm oscillations in ballistic 2D rings in gated semiconductor heterostructures [19] and square quantum dots [20].

The role of disorder is much more complicated in ballistic systems than in diffusive ones. As was shown in [21–25], the disorder-induced 'forbidden gaps' can be important also in pure systems if the external flux varies in time slowly enough. As a result, one can discriminate between *diffusive, ballistic* and *clean* devices. Clean systems are those where impurity scattering is not important. In [21–25] it was proposed that for the case of scattering by impurities with a *short-range potential* the criterion for a device to be clean is

$$\hbar/\tau_{\mathrm{el}}\Delta \ll 1. \tag{1}$$

Here Δ is the average interlevel distance while τ_{el} is the time for transport relaxation due to elastic scattering.

We want to stress here that the above classification is not universal and should be very much dependent on the phenomenon under consideration. In particular, although the criterion (1) is correct for short-range impurity potentials it becomes qualitatively wrong for scattering by a soft potential. In this case transport relaxation and forbidden gaps are produced by different scattering processes which occur with very different probabilities and hence can be distinguished from each other. Therefore the definition of a clean system in this case requires a much less stringent restriction than that given by (1).

At the present time available nanoscale structures do not meet the inequality (1). Nevertheless, we believe that quantum dots with wide enough undoped (spacer) regions produced by modulated doping do behave as if they were 'clean'.

The physical reason for this is that the impurities in these structures produce a rather smooth potential whose Fourier components decay exponentially with increase of momentum transfer. On the other hand, the forbidden gaps are determined [26] by the potential matrix elements corresponding to elastic scattering with momentum transfer $\sim p_{\mathrm{F}}$. Estimates presented below show that disorder-induced forbidden gaps in 2D ballistic dots with a typical spacer width are so small that they can be ignored for any reasonable frequency of the flux variation.

In this paper we concentrate on the magnetic response of a *clean* quantum dot to a time-dependent magnetic flux $\Phi(t)$. The case of slow flux variation, $\omega \ll \Delta/\hbar$, on the scale of the typical interlevel spacing Δ, is considered ($\omega = \dot{\Phi}/\Phi$). As a result, one can assume the electronic levels to adiabatically follow the flux variation. It will be shown that the Aharonov–Bohm (AB) oscillations are crucially dependent on the relaxation which leads to a redistribution of the electrons between the time-dependent levels driven by the flux. In the absence of relaxation, AB oscillations disappear.

The relaxation processes that might be responsible for restoration of the AB oscillations will be shown to be rather unusual—a relaxation of the induced magnetic moment requires simultaneously a large momentum transfer and a small energy transfer. Consequently, in

an isolated quantum dot, relaxation requires a very special 'inelastic backscattering' of electrons, which cannot be associated with conventional one-phonon scattering processes since they cannot fulfil the necessary energy and momentum conservation laws. As a result, the effective relaxation time τ for inelastic backscattering is large and one can expect strong non-equilibrium effects, as described briefly in [27].

The paper is organized as follows. In section 2 we formulate the problem and discuss the possibility of using an adiabatic approximation (in time) for the electron energy spectrum. We also discuss in this section criteria for Landau–Zener breakdown which destroys the impurity-produced gaps in the spectrum. In addition we discuss the physics of the non-equilibrium magnetization in the adiabatic approximation. A quantitative description is presented in section 3 where we discuss different scales for equilibrium oscillations with the flux of the magnetic moment and how this picture evolves when temperature and frequency are increased. In addition we discuss sudden switching of the magnetic flux and possible measurements of the inelastic backscattering time of electrons.

2. Qualitative discussion

2.1. Formulation of the problem

When considering a time-dependent flux one could expect a non-equilibrium behaviour of the system. The corresponding description of the kinetics depends on the relationship between the flux variation frequency ω and the internal rates which characterize both dynamical evolution and relaxation in the system. Two different time-scales for the dynamic evolution can be distinguished. The first is related to the interlevel spacing $\Delta/\hbar \approx E_F/\hbar N$ and the second to the inverse time which one needs to form a forbidden gap due to backscattering by an impurity potential.

To exhibit a pronounced non-equilibrium effect the system has to follow adiabatically the unperturbed (by the impurity potential) energy levels. In other words, the forbidden gaps have to be overcome. That can happen at high enough frequency due to Landau–Zener tunnelling. The estimates given in the next section will show that the above condition can be met at relatively low frequency. On the other hand, for $\hbar\omega \ll \Delta$ the time dependence of the flux $\Phi(t)$ does not lead to non-adiabatic transitions between the unperturbed levels. As a result, the electronic states remain coherent in a time-dependent field which leads only to an adiabatic time-dependent shift of the energy levels, keeping them sharp. If the electron distribution in energy space corresponds to equilibrium at some time t_0, in the absence of an interlevel relaxation it does not change in time leading to a monotonic increase of magnetization with the flux increase. At some later time t_1 the distribution appears strongly non-equilibrium with respect to the positions of the levels at t_1, but it is still sharp in energy space. Relaxation tends to restore the equilibrium distribution corresponding to to level positions at t_1 leading to a decrease of the magnetization. As a result, we arrive at the AB oscillations of the magnetization. It is worth emphasizing the difference between our situation and the one where the Landau–Zener tunnelling does not take place [13]. It that case, the AB oscillations exist both in the absence of relaxation and in the limit of strong relaxation.

2.2. Criteria for Landau–Zener breakdown

As has been mentioned, disorder produces subgaps in the energy spectrum due to energy level repulsion (anti-crossing) near points where the unperturbed levels are degenerate. In

the nearly-free-electron approximation, the width of the subgap near the anti-crossing point between modes m and n is equal to the matrix element of the impurity potential, $\mathcal{V}_{mn}$. The latter has to be estimated in different ways for the cases of short- and long-range potentials.

2.2.1. Short-range potential. For short-range impurities one can express the impurity potential as a sum of the contributions of different defects:

$$\mathcal{V}(\boldsymbol{r}) = \sum_i V(\boldsymbol{r} - \boldsymbol{r}_i).$$

Consequently the characteristic value of $\mathcal{V}_{mn}$ is $\mathcal{V}_{\boldsymbol{q}}/A$ where $\mathcal{V}_{\boldsymbol{q}}$ is the typical Fourier component of the total potential for q of the order of the characteristic momentum transfer between the states m and n, and A is the sample area (we have the 2D case in mind). Then, we get

$$|\mathcal{V}_{\boldsymbol{q}}|^2 \approx N_i |V_q|^2$$

where N_i is the number of impurities. As a result, the characteristic value of the subgap, g, is

$$g \approx \sqrt{\frac{1}{A} n_i |V_q|^2} \approx \sqrt{\frac{\hbar \Delta}{\tau_{\mathrm{el}}}} \tag{2}$$

where n_i is the impurity concentration, while Δ is the average interlevel distance connected to the density of states ν by $\Delta = (\nu A)^{-1}$. In a disc of radius a, $\Delta \approx E_0 = \hbar^2/2m^*a^2$.

Finally, the ratio g/Δ can be expressed as

$$\frac{g}{\Delta} \approx \sqrt{\frac{\hbar}{\tau_{\mathrm{el}} \Delta}} \ll 1. \tag{3}$$

Requiring that ratio to be small we recover the inequality (1) as a necessary condition for being allowed to ignore disorder-induced gaps in thermal properties.

Returning to the time-dependent properties, one must take into account that even if the subgaps are small, the system must be able to tunnel through them, the probability being

$$P_{\mathrm{LZ}} = \exp\left(-\frac{\pi g^2}{2\hbar(\mathrm{d}E/\mathrm{d}t)}\right). \tag{4}$$

According to equation (2), in order to make the subgaps unimportant $(1 - P_{\mathrm{LZ}}) \ll 1$ we have to require the very strong inequality

$$\omega \frac{\Phi}{\Phi_0} > \frac{\pi g^2}{4\hbar m E_0} \approx \frac{\Delta}{\tau_{\mathrm{el}} m E_0} \approx \frac{(k_{\mathrm{F}} a)^{-2}}{\tau_{\mathrm{el}}} \tag{5}$$

to be fulfilled. Combining this criterion with the one for the adiabaticity we get

$$\frac{\Delta}{\hbar} > \omega \frac{\Phi}{\Phi_0} > \frac{(k_{\mathrm{F}} a)^{-2}}{\tau_{\mathrm{el}}}. \tag{6}$$

It seems very difficult to meet these inequalities in an experimental situation, unless the samples are extremely clean.

At finite temperature, thermal phonons might also contribute to intergap transitions. To get the appropriate estimate one should compare the smearing of the spectrum caused by inelastic collisions, $\hbar/\tau_{\mathrm{in}}$ with the characteristic subgap width g. It is important that here we do not need the *inelastic backscattering* because transitions between closely placed radial

states also lead to subgap smearing. Estimating the phonon emission rate as $g^3/\hbar^3\omega_D^2$ we get the following criterion for subgap smearing:

$$\frac{\hbar\tau_{in}}{g} = \left(\frac{k_B T}{\hbar\omega_D}\right)^2 \frac{k_B T}{g} \geqslant 1.$$

It is hard to meet this condition experimentally.

2.2.2. Long-range scattering. To estimate the situation in modulated-doped structures one must at first map the Fourier components of the potential on a measurable quantity. In order to express the potential in terms of a transport relaxation time, let us consider a 2DEG plane separated from the region where donors (or acceptors) are placed with a spacer of thickness d. According to [28] the *inverse mobility* can be expressed through the Fourier component $D(q)$ of charge fluctuations as

$$\mu^{-1} = \frac{2\pi m^2 e^3}{\epsilon^2\hbar^3} \int_0^{2\pi} \frac{\exp(-2qd) D(q) F^2(q)}{(q+q_s)^2} (1-\cos\theta)\ \mathrm{d}\theta. \tag{7}$$

Here ϵ is the dielectric constant, $q = 2k_F \sin(\theta/2)$, $q_s = 2me^2/\epsilon\hbar^2$ is the 2DEG screening length, while the form factor $F(q) \sim 1$ for a typical system (see [28]). The quantity $D(q)$ is defined as the Fourier component of the impurity density fluctuations.

Rough estimates for relevant parameters show that only small angles θ are important, and one can put $k_F d \gg 1$, at $q \ll q_s$. As a result, we get

$$\mu^{-1} = \frac{\pi m^2 e^3}{\epsilon^2\hbar^3 q_s^2} \int_0^{\infty} \mathrm{e}^{-2k_F d\theta} D(k_F\theta)\, \theta^2\ \mathrm{d}\theta.$$

The product $M(\theta) = \mathrm{e}^{-2k_F d\theta} D(k_F\theta)$ is just the squared matrix element; one has to compare this quantity at the characteristic value of θ with a similar value at $\theta \approx 1$. As a result, to scale the important matrix element with $1/\tau_{tr}$ one obtains an extra factor

$$\eta = \frac{M(1)}{M(\theta_m)} \frac{1}{\theta_m^2} \qquad \theta_m^2 = \int M(\theta)\theta^2\ \mathrm{d}\theta \Big/ \int M(\theta)\ \mathrm{d}\theta\ . \tag{8}$$

The correlation function D is model dependent. We will discuss two usually accepted models [29, 30] of equilibrium (D_e) and non-equilibrium (D_{ne}), (frozen) spatial distribution of impurities. The correlation functions $D(q)$ have the forms [30]

$$D_e(q) = D_e(0) \left(\frac{2qd}{1-\exp(-2qd)}\right)^2 \tag{9}$$

$$D(q) = D_{ne}(0) \frac{1+2q_0 d}{q + q_0[1-\exp(-2qd)]}. \tag{10}$$

Here $q_0 = 2\pi n e^2/\epsilon T_0$, where n is the average impurity concentration in the doped region, while T_0 is a freezing temperature at which the impurity diffusion is frozen. According to [30], $T_0 \approx$ 100–150 K. At $n \approx 10^{-12}$ cm^{-3}, $q_0 \approx 4 \times 10^6$ cm$^{-1} \approx k_F$. Consequently, for both models we arrive at similar estimates for η (equation (8)):

$$\eta \approx (k_F d)^2 \exp(-2k_F d) \approx 2 \times 10^{-15} \qquad \text{at } k_F d = 20 \tag{11}$$

and the inequalities (6) can be rewritten as

$$\frac{\Delta}{\hbar} > \omega \frac{\Phi}{\Phi_0} > \eta \frac{(k_F a)^{-2}}{\tau_{el}}\ . \tag{12}$$

According to (11), the exponentially small value of η crucially diminishes the lower limit for the flux variation frequency. As a result, one comes back to the 'clean' picture even when the variation of the external field is slow.

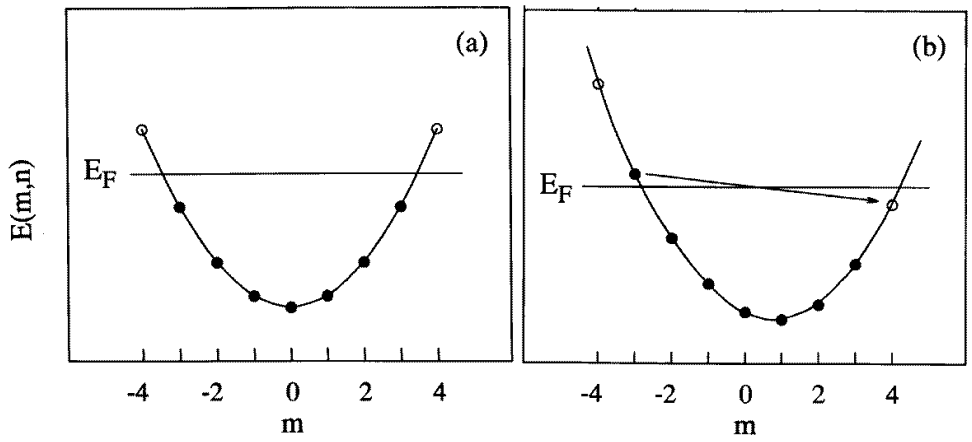

Figure 1. Schematic dependences of the energy eigenvalue $E_{m,n}$ on the magnetic quantum number m for fixed n (corresponding to a ring-shaped part of the dot): (a) occupied (filled circles) and unoccupied (empty circles) quantum states at zero magnetic field; (b) energies of the same states at finite field. To reach equilibrium the system has to to relax via inelastic backscattering events involving small energy transfer and large momentum transfer (indicated by the arrow; see the text).

2.3. The physical picture of non-equilibrium magnetization

As an illustration it is useful to first discuss electrons confined to a one-dimensional ring. The eigenenergies

$$E_m = E_0(m-\alpha)^2 \qquad \alpha = \Phi/\Phi_0 \tag{13}$$

($E_0 = \hbar^2/2m^*a^2$) of a ring are sensitive to flux and can be labelled with a magnetic quantum number $m = 0, \pm 1, \pm 2, \ldots$. In zero flux the population of $\pm m$-states is symmetric, as in figure 1(a), and there is no net magnetic moment. Increasing the flux α shifts the states with negative (positive) quantum numbers to higher (lower) energies as shown in figure 1(b), and a net magnetic moment appears. At the same time, a relaxation of the electron system becomes possible through transitions between high-energy $-m$-states and low-energy $+m$-states (figure 1(b)). The interplay between the flux-driven shift of energy levels and the 'backflow' due to relaxation determines the kinetics of the system.

Figure 1 explicitly illustrates the main features of the relaxation processes that can lead to an equilibration of the momentum distribution. First, they have to be *inelastic* to compensate for the energy mismatch that is a result of the quantization of energy levels. Secondly, it is necessary that they can provide the large momentum required to reverse the direction of the azimuthal component of the electron momentum ($-m \rightarrow +m$). We will refer to the relevant processes as 'inelastic backscattering' and emphasize that this type of scattering does not play a role in ordinary transport problems; it is special feature of our system. We will discuss possible mechanisms for inelastic backscattering later.

The situation in a quantum dot is somewhat more complicated than in a ring because of the quantization of radial as well as azimuthal motion. Nevertheless, the above qualitative discussion is still relevant; for both generation and relaxation of a magnetic moment in the dot only the azimuthal motion of the electron is of importance.

3. Quantitative calculations

3.1. The energy spectrum

In the framework of the accepted model, the electron wave function in a disc can be written as [31]

$$\psi = \mathrm{e}^{\pm \mathrm{i}m\theta}\frac{\chi(r)}{\sqrt{r}} \tag{14}$$

where $\chi(r)$ obeys the equation

$$\chi''(r) + [\alpha^2 - (m^2 - 1/2)/r^2 - \beta^2 r^2]\chi = 0. \tag{15}$$

Here r is measured in the units of a, while the eigenenergies E are related to the normalized flux α (13) as follows:

$$\alpha^2 = \frac{2mE}{\hbar^2} \mp 2\beta m \qquad \beta = \frac{eH}{2\hbar c}. \tag{16}$$

Under the conditions

$$\ell_\mathrm{c} \gg a \gg k_\mathrm{F}^{-1} \tag{17}$$

where $\ell_\mathrm{c} = v_\mathrm{F}/\omega_\mathrm{c}$ is the cyclotron radius and k_F is the Fermi wave vector [32, 33], one can treat the 'magnetic' energy $\beta^2 r^2$ as a small perturbation in comparison with the Fermi energy. By doing so one can show [31] that for states with energies of order E_F, the wave functions are confined near the surface, the energy spectrum being determined by the expression [34]

$$E_{m,n} = E_0\left[\gamma_{m,n} + 2m\alpha + \frac{\alpha^2}{3}\left(1 + \frac{2(m^2-1)}{\gamma_{m,n}^2}\right)\right]. \tag{18}$$

Here $\gamma_{m,n}$ is the nth root of the mth Bessel function, i.e. $J_m(\gamma_{m,n}) = 0$, $n = 1, 2, \ldots$ and $m = 0, \pm 1, \pm 2, \ldots$.

We can identify two different energy scales in this spectrum. The smaller scale, E_0, corresponds to the average spacing between levels in zero flux; the larger scale

$$\delta E = E_0 m \sim E_0(k_\mathrm{F} a) \tag{19}$$

is set by the characteristic shift of the energy levels when the flux is increased by Φ_0. The existence of these two energy scales results in the two different scales for the flux and temperature dependences of the induced moment already mentioned.

3.2. The quasi-static magnetic moment

When $\omega\tau \ll 1$ the induced moment can be obtained directly from the thermodynamic potential of the system. One finds

$$M_\mathrm{eq} = \frac{c}{S}\sum_{m,n} n_\mathrm{F}(E_{m,n}) M_{m,n} \tag{20}$$

where $M_{m,n}$ is the magnetic moment of a single quantum state (m, n):

$$M_{m,n} = -M_0\left[2m + \frac{2\alpha}{3}\left(1 + \frac{2(m^2-1)}{\gamma_{m,n}^2}\right)\right]. \tag{21}$$

Here $M_0 = \pi a^2 E_0/\Phi_0$ and

$$n_\mathrm{F}(E) = \left[1 + \mathrm{e}^{(E-\mu)/k_\mathrm{B}T}\right]^{-1}$$

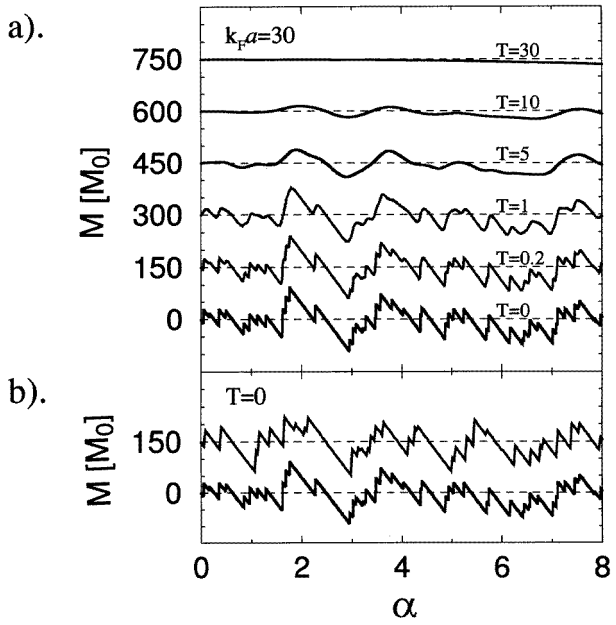

Figure 2. (a) The magnetic moment versus normalized magnetic flux $\alpha = \Phi/\Phi_0$ in a ballistic quantum dot with fixed chemical potential μ calculated at various temperatures. The units of moment, $M_0 = \pi a^2 E_0/\Phi_0$, and temperature, E_0/k_B, contain the quantum unit of flux Φ_0 and the average spacing between energy levels E_0. The fine structure in the flux dependence of the moment disappears at $T \sim 1$, whereas the Aharonov–Bohm oscillations of period unity (Φ_0) persist until $T \sim k_F a$ (a is the dot radius). (b) Zero-temperature calculations for fixed μ and for fixed number of particles give qualitatively similar results.

is the Fermi distribution function. Results at different temperatures for the magnetic moment as a function of magnetic flux for the two cases of constant chemical potential (dot connected to reservoir(s)) and constant number of particles [35] (isolated dot) are given in figure 2. We note that the flux dependences of the induced moments appear qualitatively quite similar for the dot and a metallic ring. An important quantitative difference is that the amplitude of the moment fluctuations in the 'clean' dot is of order $\pi a e v_F/c$, rather than $\pi e v_F \ell/c$ as in a 'dirty' metallic ring when $\ell \ll a$. Induced moments on this scale have been recently observed in a ballistic *ring* system [19].

3.3. The non-equilibrium response

If the time variation of the magnetic flux is not quasi-static ($\omega\tau \gtrsim 1$), essential differences appear; a non-equilibrium distribution function, $f_{m,n}(t)$, has to replace the Fermi function $n_F[E_{n,m}(\alpha(t))]$ when the moment is calculated from equation (20).

To get explicit results we restrict ourselves to the simplest case for which the relaxation time approximation applies. Here

$$\frac{\partial f_\gamma}{\partial t} = -\frac{f_\gamma(t) - n_F\big[E_\gamma(\alpha(t))\big]}{\tau_\gamma(t)} \qquad \gamma \equiv m, n. \tag{22}$$

This approximation is valid at low temperatures when electrons decay spontaneously with a lifetime $\tau_\gamma(t)$. The latter depends on the configuration of electronic levels $E_\gamma[\alpha(t)]$ and

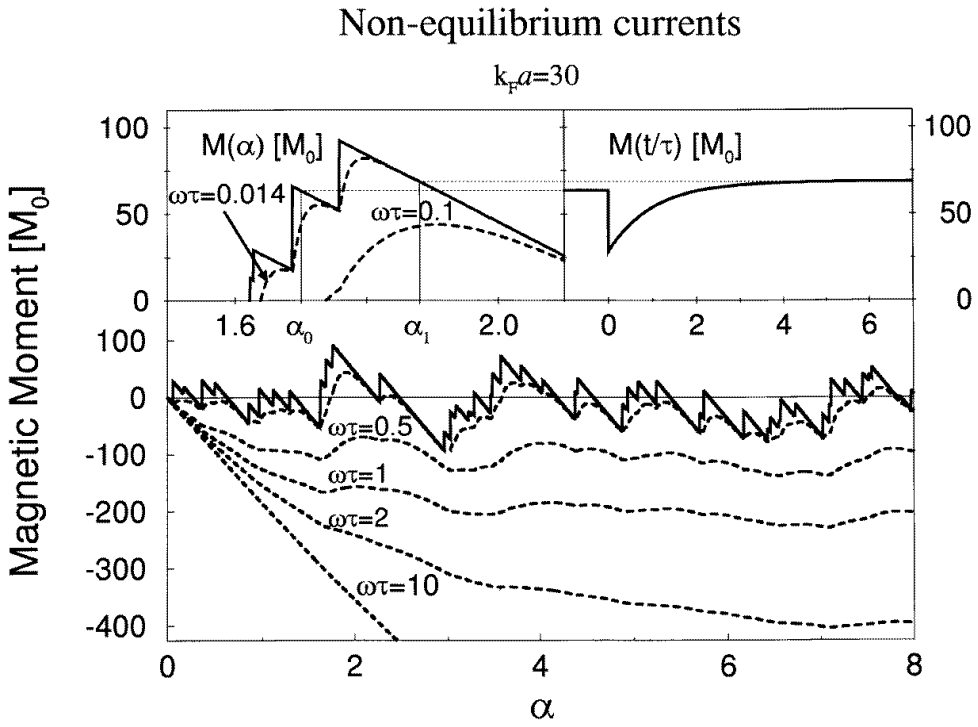

Figure 3. The magnetic moment in a ballistic quantum dot versus a normalized magnetic flux that grows linearly with time, $\alpha(t) = \omega t$, from $t = 0$. The parameter $\omega\tau$, where τ is the relaxation time, measures the rate of change of flux. The top left-hand panel shows that the fine structure in the flux dependence of the moment is smeared when $\omega\tau \sim 1/k_F a$ (a is the dot radius). For larger values of $\omega\tau$ a large diamagnetic moment is proportional to $\alpha(t) \propto t$ until it saturates at $\alpha \sim \omega\tau$. The top right-hand panel shows the current response to a sudden change of flux (cf. top left-hand panel). By monitoring how the current relaxes towards a new equilibrium value, τ could be measured.

is therefore time dependent.

The result of a numerical calculation of the induced moment for the special case where the magnetic flux increases linearly with time, $\alpha(t) = \omega t$, is shown in figure 3. For simplicity, we have assumed the relaxation time τ to be time independent. This is the case when the main source of relaxation is due to an exchange of electrons between the quantum dot and its surroundings. We believe that this approximation gives a qualitatively correct picture also in general.

Two non-equilibrium effects can be seen. Small deviations from equilibrium result in a smearing of the fine structure in the flux dependence of the moment. This happens when $\omega\tau \sim 1/k_F a$ (see the left-hand inset of figure 3). Large-scale oscillations of the induced moment, which correspond to the 'usual' Aharonov–Bohm effect with a period of the order of the flux quantum, are not affected. Drastic changes appear when the relaxation time is so large that $\omega\tau \sim 1$ or smaller. In this case the AB oscillations disappear in favour of a diamagnetic current—linearly dependent on flux (time)—until it saturates at some large flux (time). The value of the saturation moment can be readily obtained as $M_{sat} \sim N M_0 \omega\tau$. This result tells us that the saturation moment is limited only by a finite relaxation time τ and can be very large in systems where relaxation is weak. Another interesting feature

is the transient processes appearing in the case of step-like changes in the flux value. An example of such a behaviour is shown in the right-hand inset of figure 3. Here the flux is (instantaneously) changed from one value to another, for which the equilibrium-induced moment is not very different. Nevertheless, the initial diamagnetic response is quite large and decays to the new equilibrium value on the time-scale τ.

3.4. Relaxation mechanisms

A number of mechanisms can be expected to relax a non-equilibrium momentum distribution in the quantum dot. If connected to a reservoir of particles (fixed chemical potential), the reservoir acts as a sink for energetic dot electrons and a source of thermal electrons. The efficiency of this particle exchange mechanism can be characterized by a relaxation time τ_e.

Completely different relaxation mechanisms, associated with inelastic backscattering, come into play if the quantum dot is isolated (fixed number of particles). The small energy transfer and large momentum transfer called for cannot be provided by impurity or one-phonon scattering separately [36]. On the other hand, simultaneous scattering by impurities and phonons is a possible mechanism of inelastic backscattering. Multiple scattering is not described by the conventional collision operator of the Boltzmann equation, but requires an analysis that goes to higher order in the small parameter $1/k_F\ell$. Using a Green's function formalism [37], one finds an order-of-magnitude estimate for the scattering rate as $\tau_s^{-1} \approx \hbar/E_F\tau_{el}\tau_{ph}$. Here τ_{el} is the impurity relaxation time and $\tau_{ph} \approx E^3/\hbar^3\omega_D^2$ [37] is the phonon relaxation time, and ω_D is the Debye frequency, while E is the transferred energy. This energy is proportional to the magnetic flux; for a transition between the states $n, m_1 \to n, m_2$ one has $E \approx 2E_0\alpha|m_1 - m_2|$. Using these estimates and typical parameters for semiconductor nanostructures one gets $\tau_s^{-1} \approx (10^7\text{–}10^8)\alpha^3$. We stress that *any* scattering mechanism leads to a time-dependent relaxation rate because of the time dependence of E, the flux-dependent energy difference between the initial and final states. Hence, we expect inelastic backscattering, unlike the particle exchange mechanism, to cause a non-exponential relaxation of the non-equilibrium magnetic moment. The total relaxation rate can be estimated as $\tau^{-1} = \tau_e^{-1} + \tau_s^{-1}$. We believe that the relative importance of the two mechanisms can be controlled in structures where the coupling between the quantum dot and adjacent reservoirs can be varied by means of a gate voltage. Among other possible mechanisms for inelastic backscattering, one can mention inelastic scattering caused by atomic two-level systems (see [38] for a review) and by electronic two-level systems created by close pairs of filled and empty donors in a doped region of the structure (always present in semiconductor heterostructures).

4. Conclusion

In conclusion, we have shown that the criteria for discriminating between 'ballistic' and 'clean' systems suggested in [22–25] have to be reconsidered for high-mobility modulation-doped semiconductor heterostructures. As a result, the conditions for such structures to be considered 'clean' can be easily met. In addition, disorder-induced gaps in the electron spectrum can be ignored while considering dynamical properties in a time-dependent magnetic field—even one of relatively low frequency.

It is also shown that in a varying magnetic flux the magnetization of a 2D ballistic quantum dot is very sensitive to the conditions of relaxation in the system. In contrast to the usual destructive role played by inelastic scattering in mesoscopic phenomena, here inelastic scattering *restores* an Aharonov–Bohm type of quantum oscillations in the magnetization.

In the absence of such relaxation, strong non-equilibrium behaviour suppresses these oscillations in favour of large diamagnetic moments which are determined by flux rather than by the Landau level quantization as in bulk materials. A special type of inelastic backscattering is responsible for relaxation in the case of an isolated dot, and determines the maximum (saturation) value of the non-equilibrium diamagnetic moment in the case of a magnetic flux which increases linearly with time. By monitoring the transient behaviour of the induced moment as the magnetic field is switched from one value to the other one we propose that it might be possible to measure the characteristic time of inelastic backscattering estimated to be of the order of 10^{-8}–10^{-7} seconds.

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